Problem:

A scientist uses the following quadrature rule to integrate functions with fixed limits of integration.

$$\int_5^{15} f(x)dx \cong 10f(R), 5 \le R \le 15$$

If the quadrature rule is expected to give the exact value of the integral for functions of the form $ce^{3.62x}$, then find R.

Solution

The exact integral for function of the form $ce^{3.62x}$ is

$$\int_5^{15} ce^{3.62x}dx = \left[\frac{ce^{3.62x}}{3.62}\right]_5^{15}$$

$$= \frac{ce^{3.62(15)} - ce^{3.62(5)}}{3.62}$$

$$= 1.0555 \times 10^{23} c \qquad (1)$$

The quadrature formula for function of the form $ce^{3.62x}$ gives

$$10f(R) = 10(ce^{3.62R}) \qquad (2)$$

Equating what you get from integral calculus and from the quadrature formula, that is, equating equations (1) and (2), we get

$$10ce^{3.62R} = 1.0555 \times 10^{23} c$$

$$e^{3.62R} = 1.0555 \times 10^{22}$$

$$R = \frac{\ln(1.0555 \times 10^{22})}{3.62}$$

$$= \frac{50.711}{3.62}$$

$$= 14.008$$