How tall can a vertical mast be before it buckles it under own weight – a nonlinear equation model?

In his classic book, *Theory of Elastic Stability*, Timoshenko showed that one needs to solve the following nonlinear equation to find the longest length of a vertical mast before it starts to buckle under its own weight.

$$1+\sum_{n=1}^{\infty} c_n \beta^n = 0 \qquad (1)$$

where

$$c_1 = -\frac{3}{8} \qquad (2a)$$

$$c_n = -\frac{3c_{n-1}}{4n(3n-1)}, n = 2,3,\ldots \qquad (2b)$$

Once the smallest positive root, β of equation (1) is found, then the longest length of mast before which buckling would start is given by

$$L = \left(\frac{9\beta EI}{4w}\right)^{\frac{1}{3}} \qquad (3)$$

where

E = Young's modulus of elasticity,
I = second moment of area,
w = weight per unit length

Exercises

1. Since equation (1) has infinite terms, find the appropriate number of terms one should use to set up the polynomial equation to find β.
2. What is the maximum length of the hollow cylindrical mast before it buckles? Given
   Young's modulus of elasticity, $E = 210\,\text{GPa}$
   Inner diameter, $d_i = 10\,\text{cms}$
   Outer diameter, $d_0 = 15\,\text{cms}$
   Density, $\rho = 7800\,\text{kg/m}^3$