Question:

A homicide victim is found at 6:00PM in an office building that is maintained at 72˚F.

When the victim was found, his body temperature was at 85 ˚F.

Three hours later at 9:00PM, his body temperature was recorded at 78˚F.

Assume the temperature of the body at the time of death is your typical normal temperature of 98.6˚F.

The governing equation for the temperature, θ of the body is

$$\frac{d\theta}{dt} = -k(\theta - \theta_a)$$

where,

θ = temperature of the body, ˚F

θ_a = ambient temperature, ˚F

t = time, hours

k = constant based on thermal properties of the body and air.

The estimated time of death most nearly is

(A) 2:11 PM

(B) 3:13 PM

(C) 4:34 PM

(D) 5:12 PM

Solution

The correct answer is (B)

$$\frac{d\theta}{dt} = -k(\theta - \theta_a)$$

$$\frac{d\theta}{dt} + k\theta = -k\theta_a$$

The characteristic equation of the above ordinary differential equations is

$$m + k = 0$$

$$m = -k$$

$$\theta_H = Ae^{-kt}$$

The particular solution is of the form

$$\theta_p = B$$

Substituting the form of the solution in the ordinary differential equation,

$$0 + kB = k\theta_a$$

$$B = \theta_a$$

The complete solution is

$$\theta = \theta_H + \theta_p$$

$$= Ae^{-kt} + \theta_a$$

Given

$$\theta_a = 72$$

$$\theta(6) = 85$$

$$\theta(9) = 78$$

$$\theta(B) = 98.6$$

where

B = time of death

we get

$$85 = Ae^{-k6} + 72 \qquad (1)$$

$$78 = Ae^{-k9} + 72 \qquad (2)$$

$$98.6 = Ae^{-kB} + 72 \qquad (3)$$

Use equations (1) and (2) to find A and k

$$85 = Ae^{-k6} + 72$$

$$Ae^{-k6} = 13 \qquad (4)$$

$$78 = Ae^{-k9} + 72$$

$$Ae^{-k9} = 6 \qquad (5)$$

Dividing equation (5) by equation (4) gives

$$\frac{Ae^{-k6}}{Ae^{-k9}} = \frac{13}{6}$$

$$e^{3k} = 2.16667$$

$$k = 0.25773$$

Knowing the value of k, from equation (5)

A=61.028

Substitute k and A into equation (3) to find B

$$98.6 = Ae^{-kB} + 72$$
$$98.6 = 61.028e^{-0.25773B} + 72$$
$$26.6 = 61.028e^{-0.25773B}$$
$$\ln 26.6 = \ln 61.028 - 0.25773B$$
$$0.25773B = 0.83042$$
$$B = 3.2221$$

The time of death is 3.2221, that is $0.22221 \times 60 = 13.326$ minutes after 3PM.

Time of death = 3:13 PM

Note to the student:
You can also do the problem by assuming that the initial time reference is zero, and that the temperature then is $\theta(0) = 98.6^o F$. Then the temperature is given at time the body was found as $\theta(C) = 85^o F$, and that $\theta(C+3) = 78^o F$. You can now find *k, A* and *C* just like as given above. The value of *C* in fact is the time between the body was found and the time of death. You will get *C=2.777* hrs.