Solving First Order Linear ODE by Classical Solution Technique

Example

Solve the following ordinary differential equation using the classical solution technique.

$$\frac{dy}{dx} + 0.4y = 3e^{-x}\ ,\ y(0) = 5 \qquad (1)$$

Solution

The homogeneous solution for the above equation is given by

$$(D + 0.4y) = 0$$

The characteristic equation for the above equation is given by

$$r + 0.4 = 0$$

The solution to the equation is

$$r = -0.4$$

$$y_H = Ce^{-0.4x}$$

Based on the forcing function of the ordinary differential equations, the particular part of the solution is of the form Ae^{-x},

$$y_p = Ae^{-x}$$

To find A, we substitute this solution in the ordinary differential equation as

$$\frac{d(Ae^{-x})}{dx} + 0.4(Ae^{-x}) = 3e^{-x}$$

$$-Ae^{-x} + 0.4Ae^{-x} = 3e^{-x}$$

$$-0.6Ae^{-x} = 3e^{-x}$$

Comparing the coefficient of e^{-x} we have

$$-0.6A = 3$$

$$A = -5$$

Hence the particular part of the solution is

$$y_p = -5e^{-x}$$

The complete solution is given by

$$y = y_H + y_p$$

$$y = Ce^{-0.4x} - 5e^{-x}$$

The constant C is obtained by using the initial condition $y(0) = 5$

$$y(0) = Ce^{-0.4(0)} - 5e^{-(0)}$$

$$5 = C - 5$$

$$C = 5 + 5$$
$$= 10$$

The complete solution is

$$y = 10e^{-0.4x} - 5e^{-x}$$

At $x = 3$,

$$y(3) = 10e^{-0.4(3)} - 5e^{-3}$$
$$= 2.763$$