Problem Statement

Solve using 3-pt. Gauss Quadrature Rule

$$\int_{0.1}^{1.3} 5xe^{-2x} dx$$

Solution

Approach 1

The 3-point Gauss quadrature rule for

$$\int_{-1}^{1} f(x)\, dx$$

is given by

$$\int_{-1}^{1} f(x)\, \mathrm{d}x \cong c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3)$$

where

$$c_1 = 0.5556$$
$$c_2 = 0.8889$$
$$c_3 = 0.5556$$
$$x_1 = -0.7746$$
$$x_2 = 0$$
$$x_3 = 0.7746$$

$$\int_{a}^{b} f(x)\, dx = \int_{-1}^{1} \frac{b-a}{2} f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right) dx$$

$$\int_{0.1}^{1.3} f(x) dx = \int_{-1}^{1} \frac{1.3-0.1}{2} f\left(\frac{1.3-0.1}{2}x + \frac{1.3+0.1}{2}\right) dx$$

$$= \int_{-1}^{1} 0.6 f(0.6x + 0.7) dx$$

Using 3-point Guass-quadrature rule gives

$$\int_{-1}^{1} 0.6 f(0.6x + 0.7) dx$$

$$\cong [c_1 (0.6 f\ (0.6x_1 + 0.7)]$$
$$+ c_2 (0.6 f\ (0.6x_2 + 0.7))$$
$$+ c_3 (0.6 f\ (0.6x_3 + 0.7))$$

$$= [0.5556(0.6 f\ (0.6(-0.7746) + 0.7))]$$
$$+ 0.8889\ (0.6 f\ (0.6(0) + 0.7))$$
$$+ 0.5556\ (0.6 f\ (0.6(0.7746) + 0.7))$$

$$= 0.5556(0.6 f\ (0.2352))$$
$$+ 0.8889(0.6 f\ (0.7000))$$
$$+ 0.5556(-0.6 f (1.165))$$

$$= 0.5556(0.6 \times 5(0.2352) e^{-2(0.2352)})$$
$$+ 0.8889(0.6 \times 5(0.7000) e^{-2(0.7000)})$$
$$+ 0.5556(0.6 \times 5(1.165) e^{-2(1.165)})$$

$$= 0.5556 \times 0.4410 + 0.8889 \times 0.5178$$
$$+ 0.5556 \times 0.3402$$
$$= 0.8943$$

Approach 2

$$\int_a^b f(x)\,dx = \int_{-1}^{1} \frac{b-a}{2} f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)dx$$

$$\int_{0.1}^{1.3} f(x)dx = \int_{-1}^{1} \frac{1.3-0.1}{2} f\left(\frac{1.3-0.1}{2}x + \frac{1.3+0.1}{2}\right)dx$$

$$= \int_{-1}^{1} 0.6 f(0.6x + 0.7)dx$$

Since

$$f(x) = 5xe^{-2x}$$

$$\int_{-1}^{1} 0.6 f(0.6x + 0.7)dx$$

$$= \int_{-1}^{1} 0.6 \times 5(0.6x + 0.7)e^{-2(0.6x+0.7)}dx$$

$$= \int_{-1}^{1} g(x)dx$$

where

$$g(x) = 3(0.6x + 0.7)e^{-2(0.6x+0.7)}$$

The three point Gauss quadrature rule for the integral

$$\int_{-`1}^{1} g(x)\,dx$$

is given by

$$\int_{-1}^{1} g(x)\,dx \cong c_1 g(x_1) + c_2 g(x_2) + c_3 g(x_3)$$

where

$$c_1 = 0.5556$$
$$c_2 = 0.8889$$
$$c_3 = 0.5556$$
$$x_1 = -0.7746$$
$$x_2 = 0$$
$$x_3 = 0.7746$$

$$\int_{-`1}^{1} g(x)\,dx \cong c_1 g(x_1) + c_2 g(x_2) + c_3 g(x_3)$$

$$= 0.5556\left(3(0.6x_1 + 0.7)e^{-2(0.6x_1+0.7)}\right)$$
$$+ 0.8889\left(3(0.6x_2 + 0.7)e^{-2(0.6x_2+0.7)}\right)$$
$$+ 0.5556\left(3(0.6x_3 + 0.7)e^{-2(0.6x_3+0.7)}\right)$$

$$= 0.5556\left(3\left(0.6\left(-0.7746\right)+0.7\right)e^{-2\left(0.6\left(-0.7746\right)+0.7\right)}\right)$$
$$+ 0.8889\left(3\left(0.6\left(0\right)+0.7\right)e^{-2\left(0.6\left(0\right)+0.7\right)}\right)$$
$$+ 0.5556\left(3\left(0.6\left(0.7746\right)+0.7\right)e^{-2\left(0.6\left(0.7746\right)+0.7\right)}\right)$$
$$= 0.5556\times 0.4410 + 0.8889\times 0.5718$$
$$+ 0.5556\times 0.3402$$
$$= 0.8943$$